Adaptive Second Order Sliding Mode Controllers

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***Abstract*— : Three different second order sliding mode controllers are considered in this paper. An adaptive gain is implemented which adjusts the level of scalar control action on-line based on direct measurements of the equivalent control obtained by a low-pass filter. It is shown that the adaptive algorithm converge in finite time, thus the chattering is reduced in amplitude and the amount of energy demanded by the controller is reduced too. The results of a real implementation of the adaptive sliding mode controllers in a spring-mass-damper system are presented.**
Keywords: Sliding Mode, Variable Structure.

I. Introduction

The concept of adaptation in the context of control engineering refers to the variation of certain parameters with respect to a certain signal based on utilization of current information. It involves modifying the control law used by the controller in order to cope with the fact the parameters of the system being controlled are uncertain, or, to improve the performance of the controller and its effectiveness exhibiting the same dynamics properties under uncertainty conditions. Even more, adaptive control implies improving dynamic characteristics while properties of a controlled plant or environment are varying (Astrom, Wittenmark, 1989) (Shankar, Bodson, 1994).

I-A. Motivation

The main obstacle of *Sliding mode Control* (SMC) application is the *chattering* which is an oscillatory phenomenon inherent in sliding motions (see, for example, (Bartolini, Ferrara, Usai, Utkin, 2000; Boiko, Fridman, 2005; Boiko, Fridman, Pisano, Usai, 2007)). The chattering phenomenon is caused due to the high frequency switching nature of the controller. The phenomenon is well-known from literature on power converters and referred as ”*ripple*”(Perreault, Selders, 1999).

The amplitude of the chattering is proportional to the gain of the controller. In classical SMC the gain of the control should be a constant value greater than the bound of the uncertainty/perturbation. Thus the controller demands a constant amount of energy that may not be needed to maintain the system in sliding mode. Also the chattering presents a constant amplitude that can be harmful for the actuator and the plant.

I-B. Objective

The objective of this paper is to implement an adaptive gain in second order SMC in order to reduce both the chattering amplitude and the amount of energy demanded by the controller. By adapting the gain of the controller with respect to the uncertainty/perturbation, the controller demands only the amount of energy needed to compensate the uncertainty/perturbation. Hence the chattering amplitude is proportional to the amplitude of the uncertainty/perturbation and a considerable amount of energy is saved.

II. MAIN RESULT

II-A. System description

Consider the following system

$$\begin{array}{rcll} \dot{x}_i &=& x_{i-1} & i = 1, 2..., n-1 \\ \dot{x}_n &=& a(t) + b(t)u(t, x_i, x_n) & \end{array} \tag{1}$$

with the control

$$\begin{array}{rcl} u(t, x_i, x_n) &=& -k(t)w(x_i, x_n) \\ 0 < k_{min} &<& K(t) < k_{max} \end{array} \tag{2}$$

where $a(t)$ and $b(t)$ are unknown bounded functions and n is the order of the system. The terms k_{min}, k_{max} are the bounds of the minimum and maximum value of $k(t)$ respectively. The term k_{min} is introduce with the purpose of keeping the actuator always on in order to be ready to compensate any perturbation. The term k_{max} is introduce due to the limitations of the actuator which can not deliver an infinite amount of power.

The equivalent control $u_e q(t, u(t, x_i, x_n))$ is a continuous signal that is equivalent to the control signal $u(t, x_i, x_n)$ when the system is in sliding mode

$$|u_{eq}| = \left|\frac{a(t)}{b(t)}\right| = |c(t)| \tag{3}$$

The parameters of the functions $u_e q(t, u(t, x_i, x_n))$ and $u(t, x_i, x_n)$ are omitted for simplicity.

It is assume that

- A1 The uncertain functions $a(t)$ and $b(t)$ are sufficiently smooth and satisfy the following conditions:

$$\left|\frac{a(t)}{b(t)}\right| = |c(t)| < A < k_{max} \tag{4}$$

$$\frac{d|c(t)|}{dt} < L \tag{5}$$

- A2 With $k(t) > A$ the control u enforces sliding mode on some surface $\sigma(t, x_i, x_n) = 0 (\sigma \in C^1)$ (The parameters of the function $\sigma(t, x_i, x_n)$ are omitted for simplicity) with the desired properties.
- A3 The function u_{eq} is available and can be derived by filtering out a high frequency component of the discontinuous function u by a low pass filter

$$\tau \dot{u_{eq}} + u_{eq}(t, u) = u, u_{eq}(0) = 0 \tag{6}$$

with a small time constant $\tau > 0$ and the output ueq is, in fact, an estimate of $u_{eq}(t, u(t, x_i, x_n))$ satisfying

$$|y(t) - u_{eq}(t)| \leq H(\tau) \underset{\tau \to 0}{\to} 0, \tag{7}$$

II-B. *Description of the adaptive algorithm*

In (Utkin, Poznyak, Ordaz, 2011) an adaptive methodology is presented for the super-twisting algorithm. A similar methodology is presented here for second order and arbitrary order sliding mode controllers.

Consider the system (1). The adaptation law for the gain of control (2) is described as

$$\begin{aligned}
\dot{k}(t) &= \begin{cases} \gamma k_{max} + M(k(t)), & if \sigma \neq 0 \\ \gamma k(t) sign(\delta) + M(k(t)), & if \sigma = 0 \end{cases} \\
M(k(t)) &= \begin{cases} -\gamma \rho k(t), & if k(t) > k_{max} \\ \gamma \rho k(t), & if k(t) < k_{min} \\ 0, & if \;\; k_{min} \leq k(t) \leq k_{max} \end{cases} \\
\delta(t) &= \left| \frac{u_{eq}}{k(t)} \right| - \alpha
\end{aligned} \tag{8}$$

$\rho > 1$, $\gamma > \frac{L}{k_{max}}$ and $\alpha \in (0, 1)$ is the desired proportion between the unknown function $c(t)$ and gain $k(t)$. The function $M(k(t))$ is needed to ensure that the gain $k(t)$ remains bounded.

The idea of the algorithm is to increase the gain $k(t)$ until the system reach sliding mode. Once the system reach sliding mode the dynamics of the gain depends on the proportion α between the unknown function $c(t)$ and the gain $k(t)$. The gain decreases if $|u_{eq}/k(t)| < \alpha$ and increases if $|u_{eq}/k(t)| > \alpha$ until $|u_{eq}/k(t)| = \alpha \to \delta(t) = 0$.

In someway the equivalent control is an equivalent signal of the perturbation signal, thus when $\delta(t) = 0$ the proportion between the unknown function $c(t)$ and the gain $k(t)$ is α.

II-C. $\delta(t)$ *stability proof*

To show that the variable $\delta(t)$ converge to zero it is considered the following Lyapunov function

$$V(\delta(t)) = \frac{\delta(t)^2}{2} \tag{9}$$

its time derivative is calculated

$$\begin{aligned}
\dot{V}(\delta) = \delta(t)\dot{\delta}(t) &= \delta(t) \left(\frac{d|c(t)|}{dt} \frac{1}{k} - \frac{\dot{k}}{k^2} |c(t)| \right) \\
&= -\frac{1}{k} \left(\gamma |\delta(t)| |c(t)| - \frac{d|c(t)|}{dt} \delta(t) \right) \\
&\leq \frac{-|\delta(t)|}{k} \left(\gamma |c(t)| - \frac{d|c(t)|}{dt} \right) \\
&< \frac{-|\delta(t)|}{k} (\gamma |c(t)| - L) \\
&< -\frac{|\delta(t)|}{k} (\gamma k_{max} - L)
\end{aligned} \tag{10}$$

if

$$\gamma > \frac{L}{k_{max}} \tag{11}$$

$\delta(t)$ converges to zero en finite time, thus the gain reach the value

$$k(t) = \frac{|c(t)|}{\alpha} \tag{12}$$

that is a preselected minimum value that maintains the trajectories of the system in sliding mode.

III. Second order sliding mode controllers with adaptive gain

In this section the results of the implementation of three sliding mode controllers with adaptive gain in a spring-mass-damper system are presented. It is shown in figure (III) the spring-mass-damper system where the controllers were implemented. The system consist on one spring one mass and one damper. The video of each controller implemented is available at the following address *www.negrete.webs.com*.

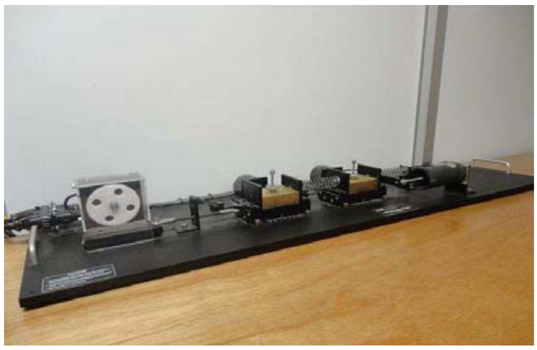

Figura 1. Spring-mass-damper system

Consider the following second order system as a model of the spring-mass-damper system

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= a(t) + b(t)u_2(t, x, y) \\ u_2 &= -k(t)w(x, y), \quad 0 < k_{min} \leq k \leq k_{max} \end{aligned} \tag{13}$$

where k_{min} and k_{max} are preselected minimum and maximum values of gain k, respectively. The functions $a(t)$ and $b(t)$ are unknown smooth bounded functions that include the parameters of the spring-mass-damper system which are assumed unknown. The state x represents the position of the mass measured in centimeters and the state y represents the derivative of the position of the mass measured in centimeter per hour. Suppose that assumptions A1, A2 and A3 holds.

III-A. Adaptive twisting control (ATWC)

Consider the system (13), where

$$w(x, y) = sign(x) + \beta sign(y), \beta \in (0, 1) \tag{14}$$

is a version of the so-called twisting algorithm.

The parameters used in the implementation are $\gamma = 4\pi$,$k_{min} = 1$,$k_{max} = 15$, $\tau = \sqrt{,001}$, $\beta = 0{,}5$, $\alpha = 0{,}45$.

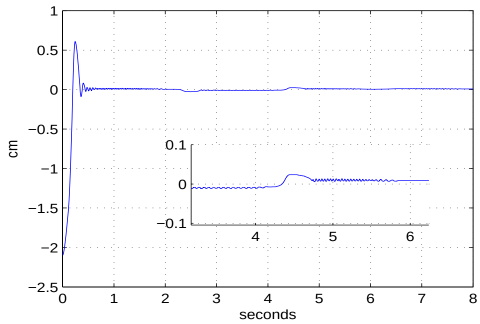

Figura 2. Position of the mass (state x)-ATWC

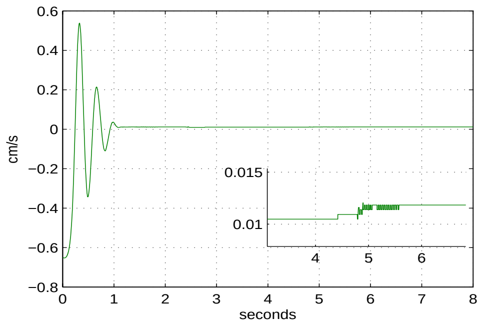

Figura 3. Velocity of the mass (state y)-ATWC

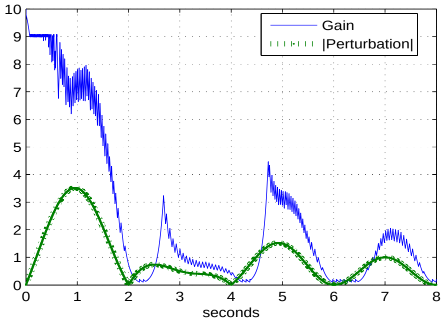

Figura 4. Gain $k(t)$ and perturbation-ATWC

III-B. Adaptive terminal control (ATEC)

Consider the system (13) where

$$w(x, y) = sign\left(y + \lambda |x|^{1/2} sign(x)\right), \lambda > 0 \tag{15}$$

is a version of the so called terminal algorithm. The values of the implementation are $\alpha = 0{,}95$, $k_{min} = 1$, $k_{max} = 15$, $\lambda = 1$, $\tau = \sqrt{,001}$

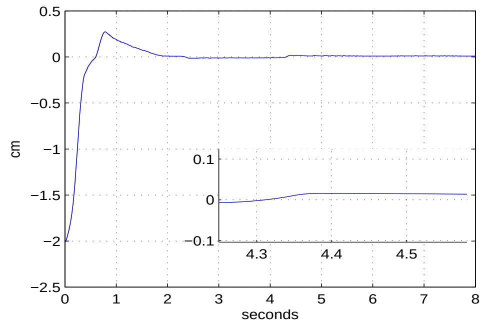

Figura 5. Position of the mass (state x)-ATEC

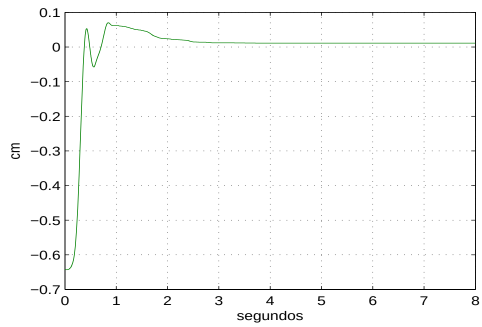

Figura 6. Velocity of the mass (state y)-ATEC

III-C. Adaptive sub-optimal control ASC

Consider the system (13) where

$$w(x, y) = \eta(t) sign(x - \beta x_m(t)) \tag{16}$$

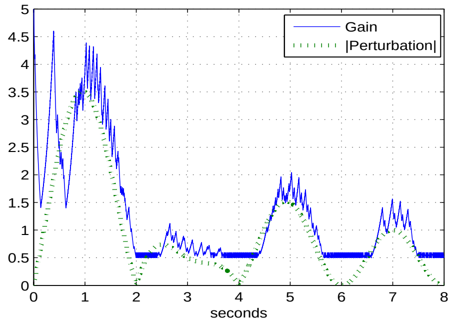

Figura 7. Gain $k(t)$ and perturbation-ATEC

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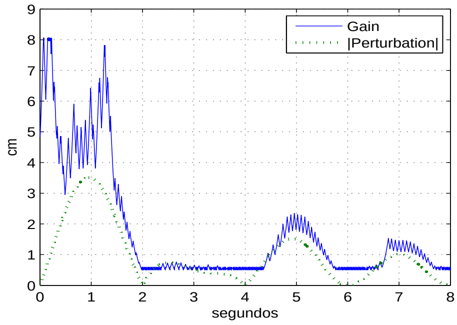

Figura 10. Gain $k(t)$ and perturbation-ASC

were

$$\eta(t) = \begin{cases} 1, & if x_m(x - \eta x_m \geq 0 \\ \eta^*, & if x_m(t)(x - \eta x_m) < 0 \end{cases} \qquad (17)$$

where $x_m(t)$ is a piece-wise function representing the value of the last singular point of x, i.e. the most recent value of x where $y = 0$.

The algorithm was implemented with the following values of parameters $\eta^* = 3$, $k_{min} = 0{,}5$, $k_{max} = 8$, $\alpha = ,9$ and $\gamma = \frac{5}{2}\pi$.

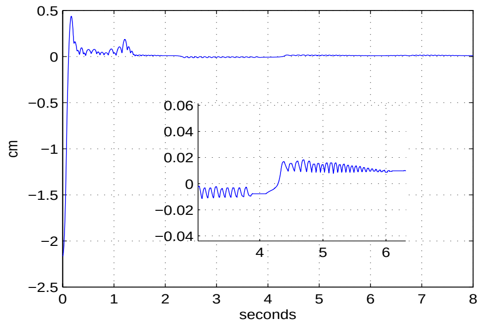

Figura 8. Position of the mass (state x)-ASC

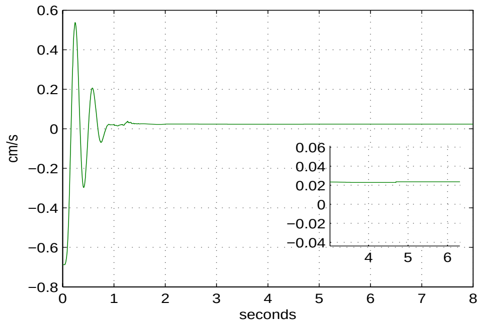

Figura 9. Velocity of the mass (state y)-ASC

III-D. Discussion of results

It is observed in figures (III-A, III-A, III-B, III-B, III-C, III-C) that the system converge to sliding mode in finite time. It is clear from the same figures that the chattering amplitude is at most of the order of micrometers and is proportional to the amplitude of the perturbation, thus the objective to reduce the amplitude chattering to a minimum level is achieve.

The dynamics of the gain of the three controllers is observed in figures (III-A, III-B, III-C). It is clear that the amplitude of the gain varies with respect to the amplitude of the perturbation. As a consequence of the adaptation of the gain, the energy demanded by the controllers is only the amount needed to compensate the perturbation saving energy unlike the classical sliding mode controllers where the gain is constant and the controllers demands a fixed amount of energy the may not be needed for the control objective.

A delay on the gain signal with respect to the perturbation is observed in figures (III-A, III-B, III-C) due to the implementation of the filter that is needed to obtain the equivalent control (u_{eq}). The delay leads to a loss of the sliding mode that is observed in figures (III-A, III-A, III-B, III-B, III-C, III-C). For example in figure (III-A) between second 4 and 5 the amplitude of the perturbation is greater than the amplitude of the gain leading to a destruction of the sliding mode that is observed in figures (III-A, III-A). This is the main disadvantage of this adaptation method.

IV. CONCLUSIONS

The implementation of an adaptive gain increases the efficiency of sliding mode controller. With the adaptive gain, the controller demands only the amount of energy necessary to compensate the perturbation. Also the chattering amplitude is reduce to a minimum value and its proportional to the amplitude of the perturbation.

The use of a low-pass filter to obtain the equivalent control produce a delay on the dynamics of the gain. The delay produce a momentary loss of the sliding mode controller. This is the main disadvantage of this method. In order to decrease the effect of the delay, a data acquisition system with a smaller sampling time is required to reduce constant of the low-pass filter, improving the accuracy of the adaptive algorithm for the gain. The smaller the filter constant , the greater the accuracy of the adaptive algorithm.

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